TOWARDS MATHEMATICAL MODELING OF ACCOUNTING FOR THE RADIATIVE CHARACTERISTICS OF SOME GROUND OBJECTS

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It is known that, in contrast to the background, in the decay processes of a number of long-lived radionuclides that constantly occur in nature, distinct temperature anomalies are created in separate parts of the earth's surface. These effects are characteristic, in particular, for significant areas contaminated with oil or ore processing wastes generated in the processes of their extraction and processing in significant areas of the majority of ore deposits. Furthermore, there are temperature anomalies caused by the transfer of energy, or the transportation of energy and, including those associated with certain types of malfunctions in the implementation of a number of types of technogenic activities. Thus, this effect is inherent both in the areas of production and primary processing of radioactive substances and in territories characterized by an increased level of contamination with heavy oil components, in the composition of which there is a constant (over time) accumulation of long-lived radionuclides.

Various optical-electronic scanning devices (Landsat 7,8 – ETM+; OLI; NOAA 18,19; Terra, Aqua –MODIS; Terra –ASTER and etc.) are used to observe temporal and spatial changes in the natural thermal radiation of the earth's surface. The interpretation of the relevant data is carried out by modern processing programs. However, the possibility of individual programs is limited only by a number of physical processes and factors affecting the final results. The programs used are based on fairly simple theoretical models built taking into account very severe restrictions and ideal meteorological and geological conditions. For the practical application of these methods in real conditions, further improvement of technical observations and theoretical analysis is required. In order for accuracy improvement of interpretation of thermal IR information on the basis of the corresponding physical properties and processes, a mathematical model of the surface thermal transfer process associated with topography has been proposed.Temperature is determined from the diffusion equation:

$$\gamma \frac{\delta^2 T}{\delta x^2} = \frac{\delta T}{\delta t} \tag{2.1}$$

wherein T=T(x,t) – the temperature at depth x from the surface; t - local time, measured from midday; γ - thermal diffusivity; ∂ - thermal conductivity.

The decision of the (2.1) for periodic heating at a given frequency ω is.

$$T(x,t) = \sum_{n=0}^{\infty} D_n \exp\left(-k\sqrt{nx}\right) \cos\left(ncot - \varepsilon_n - R\sqrt{nx}\right) \tag{2.2}$$

wherein D_n and ε_n - arbitrary coefficients.

$R = \sqrt{\frac{\omega}{2\gamma}}$ - wave number first harmonic.

Arbitrary coefficients D_n, ε_n are estimated under boundary conditions on the surface, expressed through the energy balance between the incident radiation of the Sun and the outgoing radiation of the Earth, taking into account the thermal conductivity of the corresponding environments. At the same time, the algorithms (MSD 14L; C6MODIS; C5MODIS) do not take into account atmospheric convection, thermal effects associated with the evaporation of water and the condensation of water vapor.

So, at x = 0 $$-K_{spec} \frac{\delta T(0,t)}{\delta x} = -\varepsilon \delta T^4 + I \tag{2.3}$$

Wherein I – absorbing incident radiation, composed of shortwave solar radiation (λ < 4 mkm) and longwave atmospheric radiation (λ > 4 mkm), ε – average surface emissivity, δ - Stefan–Boltzmann constant, K_{spec} – thermal conductivity.

According to (2.3), the surface temperature is directly related to the radiative characteristics of the surface. It is known that the spectral distribution of thermal radiation of an absolutely black body is described by Planck’s law:

$$W_\lambda = \frac{c_1}{\lambda^5}\frac{1}{e^{\frac{c_2}{\lambda T}}-1} \quad (2.4)$$

Wherein W_λ – radiation spectral density $vatt \cdot c^{-2} \cdot mkm^{-1}$; λ – the wavelength, mcm; T – black body temperature; C_1 and C_2 – optical coefficients.

The radiation density of real objects is always less than the density of radiation of an absolutely black body at the same temperature. The attitude of these values is called the emissivity of a real object and is determined by

$$\varepsilon = \frac{1}{\sigma T^4}\int_0^\infty \varepsilon(\lambda) W_\lambda d\lambda \quad (2.5)$$

This parameter depends on the direction of measurement. The radiation W_λ incident on the onboard sensor is approximated using the equation

$$W_\lambda = W_{\lambda\, atm}(1-\tau) + \varepsilon W_{\lambda\, earth}\tau \quad (2.6)$$

Wherein $W_{\lambda\, atm}$ – radiation of the middle layer of the atmosphere; τ - atmospheric transmittance; ε – ground emissivity; $W_{\lambda\, earth}$ – radiation leaving the earth's surface.

In order to improve the accuracy of the thermal model of the Earth's surface, the radiation of a clear sky should be taken into account, the radiation of the clouds and the scanner onboard the satellite reacts to radiation over a portion of certain wavelengths and the filter functions.Using the Laplace transform, Yacger derived the dependencies between the surface radiation and its temperature and solved the resulting equation for the surface temperature using the iterative method

$$f_n = \frac{P}{\sqrt{\pi R}}\sum_{s=1}^{m} T_s \phi_{n-s+1} \qquad n=1, 2, ..., m \quad (2.7)$$

Wherein f_n – average radiation flux incident on the earth's surface in the n spectral interval; $P = \frac{K_{spec}}{\sqrt{R}}$ – Earth's thermal inertia; τ - heating flow period; T_s - average surface temperature in the S interval; ϕ - numerical coefficients determined only by the value of m - the total number of intervals for τ. The term f_n, equal to the right side of (2.3), contains the term with T_s^4 and therefore to find the T_s must use the iterative method.

The non-linear thermal transfer problem can also be approached using the finite difference method. To ensure convergence, careful selection of spatial and temporal steps is necessary. When using the method of finite differences and Laplace transforms, the physical meaning is obscured, which can lead to excessive computer time. Therefore, it was decided to linearize a member of the equation describing the radiation flux under boundary conditions, and then check the numerical results using a more accurate solution with the Laplace transform. Within the diurnal changes in the temperature of the investigated earth's surface, the results were quite satisfactory.It is known that incident radiation I consist of shortwave solar radiation I_s and longwave radiation of the sky. The latter can be approximated σT_{sky}^4, where T_{sky}^4 – effective radiation temperature of the sky, therefore, the absorbed flux is $\varepsilon\sigma T_{sky}^4$. Then the long-wave components in the first part of the (2.3) can be linearized as

$$\varepsilon\sigma T^4 - \varepsilon\sigma T_{sky}^4 \sim 4\varepsilon\sigma T_{sky}^3\left(T - T_{sky}\right) \text{ at } \frac{T-T_{sky}}{T_{sky}} \ll 1 \quad (2.8)$$

The solution of the diffusion (2.1), which satisfies the boundary condition (2.3), the modified expression (2.8), can be obtained by simple substitution. Let us assume that

$$\phi(x,t) = T - \frac{1}{h}\frac{\delta T}{\delta x} \quad (2.9)$$

where $h = \frac{4\varepsilon\sigma T_{sky}^3}{K}$ then

$$k\frac{\delta^2\phi}{\delta x^2} = \frac{\delta\phi}{\delta t} \quad (2.10)$$

satisfies the boundary condition at x=0

$$\phi = T_{sky} + \frac{I_s}{Kh} \quad (2.11)$$

where I_s - absorbed shortwave current. This term depends on the albedo of the earth's surface A, solar declination δ, geographic latitude λ, the slope of this surface and can be expressed as

$$I_s = (1 - A)S_0 CM(Z)cosZ' \quad (2.12)$$

where S_0 - solar constant, C – coefficient taking into account the weakening of the solar flux cloud cover, $M(Z)$ - atmospheric transmission due to zenith angle, Z' - local zenith angle for incline.Atmospheric attenuation is approximately determined by law $\sqrt{secz}$;

Then

$$M(Z) = 1 - 0{,}2\sqrt{secz} \quad (2.13)$$

where secz = cosλcosδ + sinαsinδ

The local zenith angle z' can be calculated by the formula

$$cosz' = cosdcosz - sind(sin\varphi cos\delta sin\omega t - cos\varphi sin\delta cos\lambda - sin\delta sin\alpha \cos \omega t) \quad (2.14)$$

where d - angle of inclination measured from the horizon down, φ azimuth of the clockwise angle from the north direction. For convenience, in order to reduce the error associated with a regional feature, an additional parameter H (t) should be defined, which expresses local insolation:

$$H = \begin{cases} M(z)cosz', -t_R < t < t_s \\ 0 \qquad\qquad t_s < t < t_R \end{cases} \quad (2.15)$$

where t_R and t_s – time of sunset and sunrise local time, provided that$-t_R < t < t_s$, $cosz > 0$ and $cosz' > 0$. Therefore

$$I_S = (1 - A)CHS_0 \quad (2.16)$$

and the boundary condition at x = 0 expressed by (2.8) takes the form

$$\phi = T_{sky} + (1 - A)\frac{S_0 CH}{Kh} \quad (2.17)$$

The solution of equation (2.9) satisfying condition (2.17) is

$$\phi(x,t) = T_{sky} + \frac{(1 - A)S_0 C}{Kh}$$

$$\sum_{n=0}^{\infty} A_n \exp\left(-k\sqrt{nx}\right)\cos(n\omega t - \varepsilon_n - k\sqrt{nx} \quad (2.18)$$

where A_n and ε_n – amplitude and phase of the harmonic components of local insolation H.

Surface temperature can be determined by integrating (2.2)

$$T(0,t) = h\int_0^{\infty} \phi(\text{ь},t)\exp(-h\text{ь})\,d\text{ь} = T_{sky} + \frac{(1-A)S_0 C}{K}\sum_{n=0}^{\infty}\frac{A_n \cos(n\omega t - \varepsilon_n - \delta_n)}{\sqrt{(h+k\sqrt{n})^2 + (k\sqrt{n})^2}} \quad (2.19)$$

where $\delta_n = arctg(k\frac{\sqrt{n}}{n} + k\sqrt{n})$

The effect of the underground heat flux (Q) can be taken into account if we add the second solution $T = \frac{Qx}{K} + \frac{Q}{Kh}$, which satisfies the boundary condition(2.3),with a differential (2.1).

Recall that $K = \sqrt{\frac{\omega}{2k}}$; $\quad h = \frac{4\varepsilon\sigma T_{sky}}{K}$

Input $r = P\sqrt{\frac{\pi}{\tau}}$

where $\tau = \frac{2\pi}{\omega}$ and s=hK.

Then $k\sqrt{\frac{h}{n}} = r\sqrt{\frac{n}{s}}$ and $\sqrt{kn + h^2} = h\sqrt{(r\sqrt{\frac{n}{s}})^2 + 1}$,

Therefore,

$$T(0,x) = T_{sky} + \frac{Q}{s} + (1 - A)S_0 C \cdot \sum_{n=0} A_n \frac{\cos(n\omega t - \varepsilon_n - \delta_n)}{V(s + r\sqrt{n})^2 + (r + \sqrt{n})^2} \quad (2.20)$$

$$\delta_n = arctg(r\sqrt{\frac{n}{s}} + r\sqrt{n})$$

$$r = F\sqrt{\frac{\pi}{\tau}}$$

$$S = 4\varepsilon\sigma T_{sky}^3$$

Average daily temperature T_{dc} is calculated by integrating (2.19) over a daily cycle

$$T_{dc} = \frac{1}{\tau}\int_0^T T(0,t)dt = T_{sky} + \frac{Q}{S} + (1-A)S_0CA_0 \cdot cos\frac{\varepsilon_0}{S} \quad (2.21)$$

Where $$A_0 cos\varepsilon_0 = \frac{1}{\tau}\int_0^\tau H(t)dt \quad (2.22)$$

It is important to note that the T_{dc} value does not depend on the thermal inertia of the earth's surface, and together with the measured albedo values and topographic data, it can be used to estimate the subsurface heat flux Q. The difference between day and night temperatures ΔT is calculated based on the difference between midday and midnight temperatures

$$\Delta T = T(0,0) - T\left(0,\frac{\tau}{2}\right). \quad (2.23)$$

The value ΔT that is a function of thermal inertia P can be determined by systematic observations and used to calculate changes in thermal inertia.

Conclusion. Satellite information obtained in the thermal IR spectrum is used in various geological and natural fields. Since the interpretation of such information is complicated by the influence of numerous factors, the developed model makes it possible to determine the optimal observation time for obtaining quantitative characteristics of various surface properties. It is established that the ratio of the difference between day and night temperatures to the albedo value depends only on the thermal inertia, and therefore it can be used to isolate geological objects. The dependence of thermal inertia on density, water content and to some extent on the composition and condition of vegetation cover suggests that the described method will be useful for detecting and accurately predicting the incidence of agricultural crops.